Modeling of Attacking and Defending Strategies in Situations with Intentional Threats

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ABSTRACT

Intentional threats including terrorism have become a worldwide catastrophe risk since recent years. To protect the cities from being attacked, the macro-level study of decision analysis should be given more considerations. In this paper, we proposed a model for describing the strategic game between attackers and defenders based on the methodology of matrix game. This model can be employed to determine which target will be selected by attackers and which attacking strategy and defending strategy will be chosen by attackers and defenders respectively. Furthermore, the defenders of the city can use this model to set priorities among their defending strategies. The importance of this work is to establish a reasonable framework for modeling the attacking and defending strategies rather than assessing the real risk of urban targets, so the model is illustrated by using fictitious numbers. The model proposed in this paper can provide scientific basis for macroscopic decision making in responding to intentional threats.

Keywords

Intentional threats, decision analysis, matrix game, target, strategy.

INTRODUCTION

Recent years have seen frequent occurrences of intentional threats, like terrorism events and sabotages, which have become a worldwide catastrophe risk. For example, 9·11 Terrorist Attacks in the USA, 3·11 Madrid Train Bombings in Spain, Lhasa 3·14 vandalism and arson in China and 1·24 Moscow Airport Bombing were all typical intentional attacks and led to huge losses and fatalities. Thus, more and more researchers around the world are paying close attention to the study on the risk analysis of intentional attacks.

Unlike natural disasters, intentional attacks are induced by intentional attackers who usually select attacking targets and attacking strategies based on the type of targets and the situation of urban defense strategies. Therefore, intentional attacks cannot be considered as simple random events. The Department of Homeland Security (DHS) released a risk-based performance standard for the security of chemical facilities in the United States, which has already been generally accepted. However, the rationality of attackers has not been considered among the process of risk assessment.

In order to model the intentional threats considering the rationality of attackers and defenders, a series of researches have been performed.

Engineering risk analysis method has been used to establish a probabilistic model to describe the process of decision-making between attackers and defenders and furthermore to set priorities among their strategies (Paté-Cornell, 2002). As another important attempt, game theory is introduced into the study on target selection of intentional attackers, which provided an effective method of solving the problems of decision rationality and strategic interaction in the process of target selection (Hausken, 2008; Hausken and Levitin, 2009). Moreover, in order to propose more realistic game theoretical framework for the decision-making analysis, model taking into account of both the success probability of an attack and the value of the target has been established (Bier et al., 2005) where attackers may choose a target which is most vulnerable instead of one that would cause the highest expected damage. In addition, another research showed that the probability of the attacker choosing a particular target is inversely proportional to the marginal effectiveness of defense at that target (Woo, 2002).

In the process of unban defense from intentional threats, the defenders should allocate resources or investments reasonably to the potential target in the city to protect the targets from being attacked or to minimize the expected loss. In this paper, taking into account of the features of intentional attacks, we proposed a model for describing the strategic game between attackers and defenders based on the methodology of matrix games. This model can be employed to determine which target will be selected by attackers and which attacking strategy and defending strategy will be chosen by attackers and defenders respectively. Furthermore, the defenders of the city can use this model to set priorities among their defending strategies. Therefore, it can provide the planner and decision makers of urban defense engineering with scientific basis for macroscopic decision making in responding to intentional threats.

THE OVERARCHING MODEL

Under the conditions of intentional threats, attackers can choose different potential targets such as infrastructure systems and networks, government buildings, symbolic buildings and even urban population. For the same target, attackers can choose more than one attacking strategy to achieve their goals. Based on the data provided by Global Terrorism Database (GTD), the means of terrorist attacks often include: explosives attack, firearms attack, incendiary attack, chemical attack, biological attack, radiological attack et al.

According to different targets, defenders should assess their vulnerabilities comprehensively and choose reasonable defense strategies, including improving the security planning on urban infrastructure, establishing security surveillance or early warning systems, and enhancing the patrol, et.al. . However, the resources and investments spent on strengthening the security of these targets are limited. Therefore, it is necessary to set priorities among the targets and choose the most effective strategies to protect these targets.

The overarching model is based on the game-theoretical framework and takes account of strategies of both attackers and defenders on different targets. Corresponding diagram representation is shown in Figure 1.

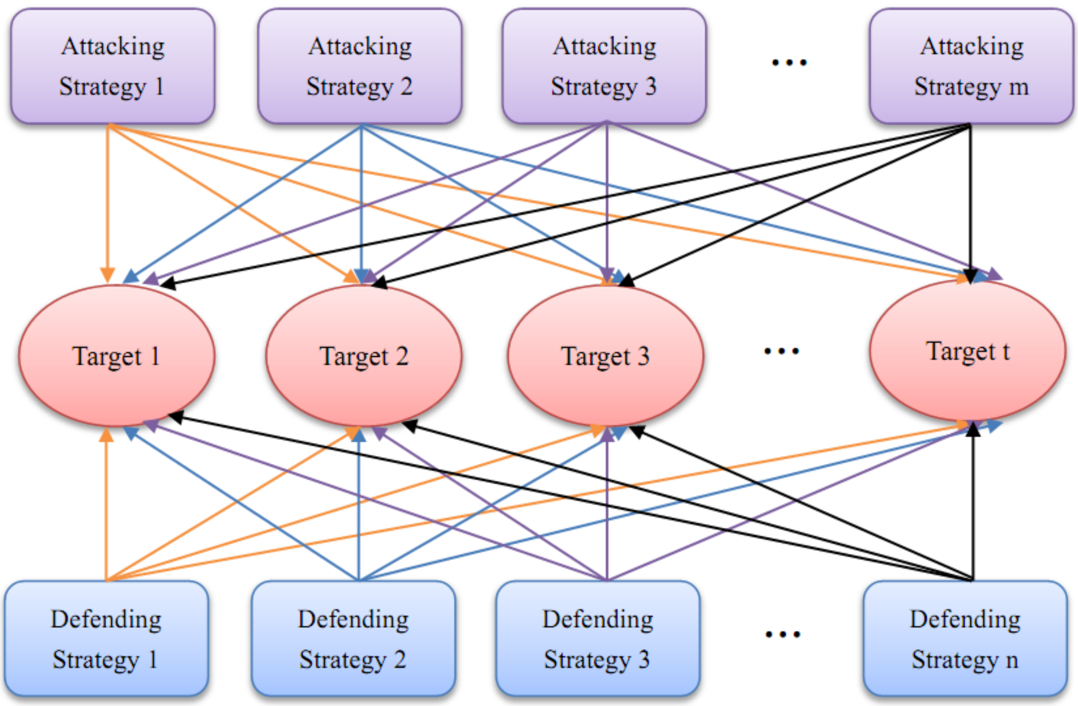

Figure 1. Diagram representation of the overarching model

The objective of the model is to use the Matrix Game approach to identify:

- The most attractive target and weapon type to the attackers

• The most effective defending strategies of reducing the overall risk

In this model, we assume that all attackers and defenders obey the axioms of the Expected Utility Theory which means that they will take the actions which will maximize their utility. The utilities of attackers can usually be understood as casualties or property losses caused by the events, which, in the model, can simply be defined as the expected loss. Meanwhile, the utilities of defenders can be defined as the negative of expected loss, which is reasonable since the game in situations with intentional threats is non-cooperative. In this model, a participant's gain or loss is exactly balanced by the loss or gain of the other, rendering a zero-sum overall utilities. Then we have,

$$u_a = -u_d \tag{1}$$

where u_a is the utility of attackers and u_d is the utility of the defenders.

Based on the above assumptions, the overarching model is established.

The definition of the strategy set

$A=\{a_{ki} \mid k \in T, i \in M\}$ is the attacking strategy set, where $T=\{1,2,\cdots,t\}$ represents t different targets and $M=\{1,2,\cdots,m\}$ represents m different attacking strategies. $D=\{d_{lj} \mid l \in T, j \in N\}$ is the defending strategy set, where $N=\{1,2,\cdots,n\}$ represents n different defending strategies. $S=\{s_{kilj} \mid k \in T, l \in T, i \in M, j \in N\}$ is the state set, where each state corresponds to an attacking strategy and a defending strategy on target k and l respectively.

The establishment of matrix game G

$G=(A, D; U_a, U_d)$, where U_a is the utility matrix of attackers, U_d is the utility matrix of defenders:

$$U_a = \begin{bmatrix} u^a_{1111} & u^a_{1112} & \cdots & u^a_{11lj} & \cdots & u^a_{11tn} \\ u^a_{1211} & u^a_{1212} & \cdots & u^a_{12lj} & \cdots & u^a_{12tn} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ u^a_{ki11} & u^a_{ki12} & \cdots & u^a_{kilj} & \cdots & u^a_{kitn} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ u^a_{tm11} & u^a_{tm12} & \cdots & u^a_{tmlj} & \cdots & u^a_{tmtn} \end{bmatrix} \tag{2}$$

$$U_d = \begin{bmatrix} u^d_{1111} & u^d_{1112} & \cdots & u^d_{11lj} & \cdots & u^d_{11tn} \\ u^d_{1211} & u^d_{1212} & \cdots & u^d_{12lj} & \cdots & u^d_{12tn} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ u^d_{ki11} & u^d_{ki12} & \cdots & u^d_{kilj} & \cdots & u^d_{kitn} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ u^d_{tm11} & u^d_{tm12} & \cdots & u^d_{tmlj} & \cdots & u^d_{tmtn} \end{bmatrix} \tag{3}$$

Based on the assumptions of the zero-sum game, the following relation exists between the utility matrices,

$$U_d = -U_a \tag{4}$$

Each element in the matrix corresponds to a strategy state s, which describes the utility of the attacker or defender in that state. In the utility matrix,

$$u_{kilj} = f(V_{kilj}, C_{kilj}) \tag{5}$$

where V_{kilj} is the function of the success probabilities of the event, C_{kilj} is the function of the consequences when the event occurs and causes losses. Utility *u* is the value of the expected loss determined by the above functions. The utilities of attackers are defined as the expected loss in the event. Therefore, the utility *u* can be defined as V ×C:

$$u_{kilj} = V_{kilj} \cdot C_{kilj} \tag{6}$$

THE CONTEST SUCCESS FUNCTION

Different from natural disasters, the events with intentional threats have the feature of risk deflection, which means attackers can adjust their potential targets to find the best strategy which can balance costs and benefits well. And that strategy may not be consistent with investment focus of defenders. Attackers can choose the target with weak defense to avoid high risk. In order to reduce the risk, attackers are likely to give up high-value targets and choose the high-vulnerability target considering the success probability. Therefore, the success probability for the attack is essential to be considered.

The function of the success probability of the event usually has two types of expression: proportional (Hirshleifer, 1989; Skaperdas, 1996) and exponential (Bier et al., 2008). The research on the reliability of the system (Hausken, 2008) proved that the proportional function can express the opposability more clearly. In this model we use the proportional function to describe the success probability of the event. We define the probability of a successful attack on target *k* asset

$$V_{kilj} = \frac{y_i^{\alpha}}{(\beta_{lj} \cdot x_k)^{\alpha} + y_i^{\alpha}} \tag{7}$$

where V_{kilj} is the success probability of the attack event, y_i is the attacking intensity of type *i* attacks, x_k is the defending intensity of target *k*, α expresses the intensity of the contest and β is the coefficient of defending intensity.

When $\alpha = 0$, x_k and y_i have the same impact (50%) on V_{kilj}. When $0 < \alpha < 1$, it gives a disproportional advantage of intensity less than one's opponent. When $\alpha = 1$, x_k and y_i have proportional impact on V_{kilj}. When $\alpha > 1$, it gives a disproportional advantage of intensity more than one's opponent. Finally, $\alpha = \infty$ gives a step function where ''winner-takes-all'' (Levitin and Hausken, 2010). Generally, the value of α is lower for systems which are more difficult to attack and higher for those that are easier to attack (Hirshleifer, 1995). For a general system, $\alpha > 1$ is the appropriate value (Hausken, 2008).

The coefficient β_{lj} represents the defense-strengthening impact of type *j* defending strategy on target *k*. We assume that the targets are independent, so when attacker and defender pay attention to different targets, there will be no defense-strengthening impact on target *k*. Then we have,

$$\beta_{lj} = \begin{cases} \beta_j, & l = k \\ 1, & l \neq k \end{cases} \tag{8}$$

For general circumstance, $\beta_j > 1$ is appropriate.

CONSEQUENCE OF THE EVENT

The purpose of attacking under the situation of intentional threats is to make the maximum casualties, economic losses, social impacts or other negative consequences.

The attacker or defender's estimation basis for assessing the consequence *C* of the event may differ, since it is related to their subjective motives. Based on that, the same event may bring different level of consequences according to the agents' different assessments. For instance, when the attackers intend to cause casualties, the casualties will be more actractive to them than other factors like social impacts. Generally, the agents' estimation basis for assessing the consequence *C* of the event is closely related to the value of targets.

The value of targets could be measured by loss of life, primary economic loss, national economic stress and inconvenience, decrease presence considered undesirable by an attacker, increase presence considered desirable by an attacker, opportunity to leverage with other terrorists (Beitel, Gertman, and Plum, 2004). Hausken pointed out that the total value of a target has to be measured by economic value, human value and symbolic value (Hausken, 2011). Based on Hausken's work, we can also define the consequence *C* as follows,

$$C_{kilj} = \gamma_{lj}^{E} \cdot \lambda_i^{E} \cdot e_k + \gamma_{lj}^{H} \cdot \lambda_i^{H} \cdot h_k + \gamma_{lj}^{S} \cdot \lambda_i^{S} \cdot s_k \quad (9)$$

where e_k is the economic value of target *k*, h_k is the human value and s_k is the symbolic value. λ_i is the coefficient of attacking effect of strategy *i*, which represents the level of loss, so we have $0 < \lambda_i < 1$. γ_{lj} is the coefficient of defending effect of type *j* defending strategy on target *k*. We assume that the targets are independent, so when attacker and defender pay attention to different targets, there will be no defending effect on target *k*. Then we have,

$$\gamma_{lj} = \begin{cases} \gamma_j, & l = k \\ 1, & l \neq k \end{cases} \quad (10)$$

Generally, $0 < \gamma_{lj} < 1$ is appropriate.

GAME SOLUTIONS UNDER THE ASSUMPTION OF PERFECT RATIONALITY

In the perfect rationality model, attackers will choose the strategy whose utility is the largest. If the pure strategy (a_{ki}^*, d_{lj}^*) exists, such that for any *ki* and *lj,* we have $u_{(ki)(lj)*} \le u_{(ki)*(lj)*} \le u_{(ki)*(lj)}$. And then (a_{ki}^*, d_{lj}^*) is the solution of *G* under the pure strategy and $u_{(ki)*(lj)*}$ is called the equilibrium value. At the equilibrium state $s_{(ki)*(lj)*}$, the utility of the attacker will be not less than $u_{(ki)*(lj)*}$, while the loss of the defender will be not more than $u_{(ki)*(lj)*}$. a_{ki}^* and d_{lj}^* are respectively called the attacker's and the defender's optimal pure strategies. We have the utilities of the attacker and the defender,

$$\begin{cases} u_a = u_{(ki)*(lj)*} \\ u_d = -u_{(ki)*(lj)*} \end{cases} \quad (11)$$

In most cases, the game does not have solution in the sense of pure strategy. Thus we should find the game's solution under the mixed strategy (P, Q) and give a probability distribution of different strategies.

$$\begin{aligned} P &= (p \in E^{m\times t} \mid 0 \le p_{ki} \le 1, \textstyle\sum_{k=1}^{t} \sum_{i=1}^{m} p_i = 1) \\ Q &= (q \in E^{n\times t} \mid 0 \le q_{lj} \le 1, \textstyle\sum_{l=1}^{t} \sum_{j=1}^{n} q_j = 1) \end{aligned} \quad (12)$$

where $E^{m\times t}$ and $E^{n\times t}$ are the m×t-dimensional and n×t-dimensional Euclid space respectively.

The utility of the attacker is:

$$u_a = E = \sum_{k=1}^{t} \sum_{l=1}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} u_{kilj} p_{ki} q_{lj} = p \cdot U_a \cdot q \quad (13)$$

If there is an attacker's strategy p^* and a defender's strategy q^* which satisfy the equation (14), (p^*, q^*) is the solution of the game G, called the optimal mixed strategy.

$$\max_{p\in P} E(p, q^*) = \min_{q\in Q} E(p^*, q) \quad (14)$$

Under the assumption of perfect rationality, we can use the linear programming method to obtain the solution. V_1 is the attacker's value of the game and V_2 is the defender's value of the game. Generally, the attacker's benefits will not be more than the defender's loss. So we have:

$$V_1 \le V_G \le V_2 \quad (15)$$

where V_G is the solution of the game. Solving V_G is equivalent to solving the following linear programming problem.

$$\begin{cases} \min V_2 \\ s.t. \quad \sum_k \sum_i u_{kilj} p_{ki} \geq V_2 \\ \sum_k \sum_i p_{ki} = 1 \\ p_{ki} \geq 0 \end{cases} \tag{16}$$

$$\begin{cases} \max V_1 \\ s.t. \quad \sum_l \sum_j u_{kilj} q_{lj} \leq V_1 \\ \sum_l \sum_j q_{lj} = 1 \\ q_{lj} \geq 0 \end{cases} \tag{17}$$

Set $\varepsilon_{ki} = p_{ki} / V_1$, $\omega_{lj} = q_{lj} / V_2$, and the solution of the game is $V_G = \dfrac{1}{\sum_k \sum_i \varepsilon_{ki}}$. Then the linear programming will be transformed into the form as follows:

$$\begin{cases} \min \sum_k \sum_i \varepsilon_{ki} \\ s.t. \sum_k \sum_i u_{kilj} \varepsilon_{ki} \geq 1 \\ \varepsilon_{ki} \geq 0 \end{cases} \tag{18}$$

$$\begin{cases} \max \sum_l \sum_j \omega_{lj} \\ s.t. \sum_l \sum_j u_{kilj} \omega_{lj} \leq 1 \\ \omega_{lj} \geq 0 \end{cases} \tag{19}$$

The model proposed in this paper is based on the assumption of perfect rationality, which assumes that the rational agents will always take the actions which aims for maximizing their benefits respectively.

ILLUSTRATION OF THE MODEL BY USING FICTITIOUS NUMBERS

We present an illustration under the assumption of perfect rationality by using fictitious numbers. The modeling and solving under the assumption of bounded rationality is more complex, and will be developed in future work.

In this paper we consider two types of urban targets: Government Building and Infrastructure. And the urban targets may suffer the following two types of intentional attacks: Conventional Explosive and Biological Weapon. At the same time, we assume that the defenders can choose two types of defending strategies including Stepping up Patrols and Enhancing System Reliability. The strategy sets are described in Table 1.

Strategy Set		Description
Attacking Strategy	i = 1	Conventional Explosive
	i = 2	Biological Weapon
Defending Strategy	j = 1	Stepping up Patrols
	j = 2	Enhancing System Reliability
Target	k (or l) = 1	Government Building

	k (or l) = 2	Infrastructure

Table 1. Description of the strategy set

The input parameters of utility matrix can be determined by expert experience or historical statistics. The input parameters used in this paper are fictitious as shown in Table 2.

Input parameters	y_i	λ_i^E	λ_i^H	λ_i^S
i = 1	5	0.6	1	0.8
i = 2	3	1	0.9	0.9
Input parameters	β_j	γ_i^E	γ_i^H	γ_i^S
j = 1	1.3	1	1	1
j = 2	1.2	0.8	0.8	0.8
Input parameters	x_k	e_k	h_k	s_k
k = 1	5	50	60	100
k = 2	4	100	90	50
Input parameters	$\alpha = 2$			

Table 2. Input parameters

According to equation (7) and (9), we can gain the attacker's utility matrix as follow:

$$U_a = \begin{bmatrix} 63.2 & 55.7 & 85 & 85 \\ 34.1 & 31.0 & 51.4 & 51.4 \\ 115.9 & 115.9 & 91.3 & 79.1 \\ 81.4 & 81.4 & 56.4 & 50.8 \end{bmatrix} \tag{20}$$

From the utility matrix (20), we can find that the game doesn't have solutions in the sense of pure strategy. Thus we should use linear programming method to find the solution under the mixed strategy. Based on that, by using the equation (18) and (19) on software ***lingo***, we have:

$$\begin{cases} \varepsilon = (0.00676, 0, 0.00538, 0) \\ \omega = (0, 0.00108, 0, 0.0111) \\ V_G = \dfrac{1}{\sum_k \sum_i \varepsilon_{ki}} = \dfrac{1}{0.01214} = 82.385 \\ p^* = (0.557, 0, 0.443, 0) \\ q^* = (0, 0.0893, 0, 0.911) \end{cases} \tag{21}$$

The result (21) shows that the attackers will not use Biological Weapon to achieve their goals, but will choose the strategy of Conventional Explosive to attack, which is consistent with the statistical data provided in GTD to some extent. On the other hand, the defenders will not choose Stepping up Patrols as the defending strategy, but will take account of Enhancing System Reliability to minimize the expected loss. Moreover, the result (21) also shows that the defenders are more likely to enhance the reliability of the Infrastructure than that of the Government Building. The probability distributions of attacking strategies and defending strategies are shown in Figure 2 and Figure 3 respectively.

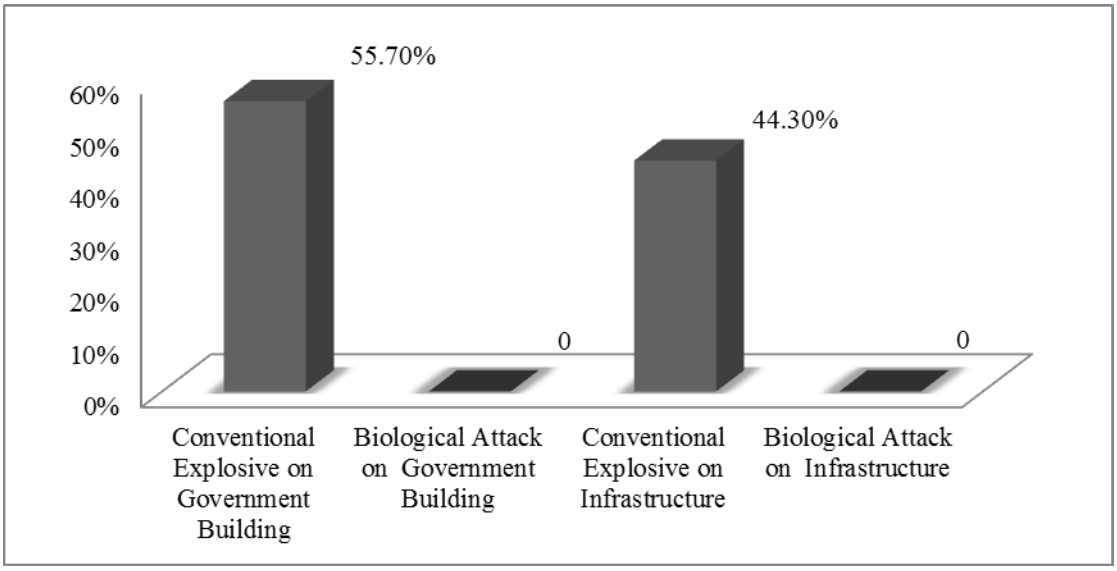

Figure 2. Probability distribution of attacking strategies

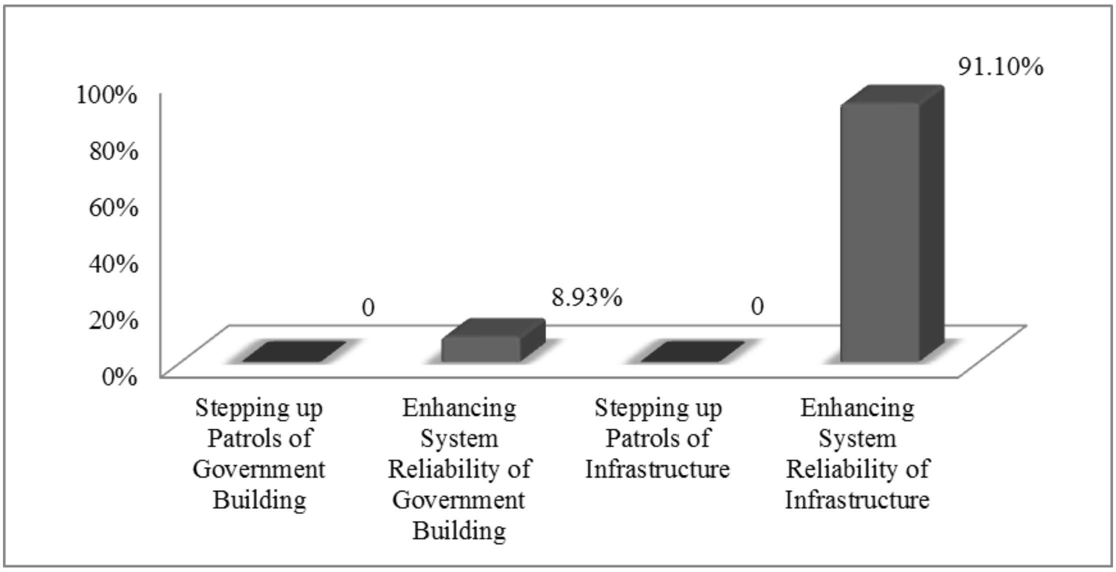

Figure 3 Probability distribution of defending strategies

CONCLUSION

In this paper, we developed a model to describe the strategic game between attackers and defenders based on the methodology of matrix games in the situation with intentional threats. In this model, we assume that attackers and defenders will always take the actions which will maximize their utility. And the utility here is defined as the product of the success probability of the attack and the consequences of the event. Moreover we proposed a method to gain the game solution of this model under the assumption of perfect rationality.

The most important purpose of this paper is to establish a reasonable framework for modeling the attacking and defending strategies rather than assessing the real risk of urban targets. Moreover, the model is illustrated by using fictitious numbers. The results prove that this model can be used to find out which urban target is more likely to be attacked, and to determine how to set priorities among the defending strategies based on the Expected Utility Theory and the Game Theory.

In the future work, we will consider more complicated situations and establish more realistic model under some specific intentional threats, and use real data to validate the model. In addition, we also plan to study on the modeling of intentional threats under the assumption of bounded rationality. Then we can provide the planner and decision makers with more scientific basis for macroscopic decision making in responding to intentional threats.

ACKNOWLEDGMENTS

This research is supported by National Nature Science Foundation of China (Grant No. 91024016, 91024032 and 70973063).

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